

# Estimation and Inference of Impulse Responses by Local Projections Òscar Jordà (2005) Summary

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## 1 Introduction

The following essay intends to present an easy-to-grasp summary of Jordà's 2005 paper of Estimation and Inference of Impulse Responses by Local Projections. You will also find Stata codes that replicate the results of application illustrated in the paper.

## 2 Describe the problem the enclosed paper solves in the literature and discuss its main contributions.

### 2.1 What is impulse response function?

Impulse response function is useful to investigate relationship between two variables in a higher dimensional system. [4] In other words, impulse response function helps us to understand how one variable will respond to shock of another variable over time.

An example of application is how inflation will respond to a shock in interest rate over a time horizon of 12 quarters.

### 2.2 Problem to solve

Impulse response function is traditionally estimated by the structural vector autoregressive (SVAR) approach. Within a VAR(p) model, the impulse responses are determined by the VAR coefficients.

These are the key drawbacks of this estimator:

1. This estimator is a highly nonlinear function of the VAR coefficient estimators. Therefore, standard errors for impulse responses from VARs are complicated as they are highly nonlinear functions of estimated parameters.[2]
2. If the true data generating process (DGP) is not VAR(p), then there could be misspecification bias.
3. VARs approximate the data globally: best, linear, one-step ahead predictors but impulse responses are functions of multi-step forecasts.[2]

### 2.3 Main contribution

Jordà observed impulse response can be estimated by local projections using simple least squares.

He argues these are the advantages of local projections:

1. They can be estimated by single-equation OLS with standard regression packages.
2. They provide simple, analytic, joint inference for impulse response coefficients.

3. They are more robust to misspecification as they are linear estimators though this is more of an intuition.
4. Experimentation with very nonlinear and flexible models is straight-forward.[2]

### 3 Summarize the proposed procedure as an algorithm and discuss its implementation. If possible, explain how it differs from other approaches in the literature.

#### 3.1 This is a summary of the proposed procedure of local projection. Please refer to Application.do file for detailed algorithm.

1. Decide on the impulse response horizon,  $h$ . In other words, how far ahead we want to predict.
2. Order data according to time variable in ascending order.
3. Generate  $t+1, \dots, t+h$  variables for all variables  $t$ .
4. Decide on the number of lags  $p$ .
5. Run  $h$ -step predictive VAR( $p$ ) that can be estimated by least squares.

This is the main estimation equation.

$$y_{t+h} = \alpha_h + \beta_h s_t + \gamma_h \mathbf{x}_t + v_{t+h}$$

$\alpha_h$  denotes regression constant,  $s_t$  is the identified shock variable, and  $\mathbf{x}_t$  denotes a vector of control variables. The coefficient  $\beta_h$  is the response of  $y$  at time  $t+h$  to the shock variable  $s_t$  at time  $t$ . The impulse responses are the sequence of all estimated  $\beta_h$ .

Impulse response function is defined as

$$\mathcal{R}(h) = E[y_{t+h}|s_t = s_1; \mathbf{x}_t] - E[y_{t+h}|s_t = s_0; \mathbf{x}_t] = \beta_h(s_1 - s_0)$$

For example, if we have 3 variables, say output ( $y$ ), inflation ( $infl$ ) and interest ( $inte$ ) rate, pick lags  $p = 2$  and time horizon  $h = 12$  quarters.

So for horizon  $h$ , the response of output to a shock of inflation will be estimated as below

$$y_{t+h} = \alpha_h + \beta_h infl_t + \beta_h^1 infl_{t-1} + \beta_h^2 infl_{t-2} + \gamma_h^1 y_{t-1} + \gamma_h^2 y_{t-2} + \tau_h^1 inte_{t-1} + \tau_h^2 inte_{t-2} + u_{t+h}$$

$h = 0, \dots, 12$

So we will run this equation 13 times as we are interested in time horizon  $h = 12$  quarters.

6. Use Newey-West estimator for standard error to construct confidence interval as it does not require VAR( $p$ ) to be correctly specified.[1]

#### 3.2 Main Difference

The main contrast to estimation of impulse response function using VAR( $p$ ) approach is that the impulse responses are determined by the VAR coefficients. In other words, impulse response matrices can be written as a simple recursion in the VAR coefficients.[1] Local projections, however, provides a straightforward linear estimator for impulse response.

According to Jordà, for a long time, many researchers specify a VAR just to compute dynamic multipliers like impulse response function, even though VAR is not of interest.

### 3.3 Discuss their empirical findings.

Jordà applied local projections to investigate the dynamic properties of inflation-output trade-offs. The paper further investigate nonlinearities in the impulse responses by using Lagrange multiplier (LM) test of Bruce Hansen (2000) for a threshold.

In this empirical application, it finds that inflation and output are far more responsive to interest rates in the low-inflation regime than in the high-inflation regime. Moreover, it supports observation that adverse inflation-unemployment outcomes of the 1970s were not the result of bad policy but the result of a changing economic environment.

### 3.4 Comment on any limitation of the framework or the empirical findings.

These are the limitations of the discussion in this paper.

1. Jordà argued in this paper that local projection estimator is less sensitive to misspecification as it is a straightforward linear estimator. This is, however, intuitive and hence unclear.
2. This paper did not discuss the identification of matrix  $D$  which contains columns  $d_i$  that contain relevant experimental shocks. Jordà thinks this is important because one, statistical-based structural identification of contemporaneous causal links is hard to find. Second, a one-time shock to a given variable in the system may not be the only type of experiment of interest.
3. This paper did not discuss the bias-variance trade-off between local projections and VAR. From Li, Plagborg-Møller & Wolf (2024) found that if bias is a major concern, then researchers should prefer least-square local projections to VAR. [3] Plagborg-Møller & Wolf (2021) suggests that LP and VAR share same population impulse response function if  $h_j=p$ . In fact, there is no meaningful trade-off if interest centers on short horizon or very large lag length.

## References

- [1] Bruce Hansen. *Econometrics*. Princeton University Press, 2022.
- [2] Òscar Jordà. Impulse responses by local projections: Practical issues. <https://www.eco.uc3m.es/~jgonzalo/teaching/PhDTimeSeries/Local%20Projections%20Jorda.pdf>. [Accessed 26-07-2024].
- [3] Dake Li, Mikkel Plagborg-Møller, and Christian K Wolf. Local projections vs. vars: Lessons from thousands of dgps. *Journal of Econometrics*, page 105722, 2024.
- [4] Helmut Lütkepohl. *New introduction to multiple time series analysis*. Springer Science & Business Media, 2005.